UDC 62-50

ON THE CORRECTION OF A NONLINEAR CONTROLLED PROCESS

PMM Vol. 41, № 2, 1977, pp. 210-218

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(Received March 18, 1976)

A constructive method is proposed for correcting a process described by a nonlinear vector differential equation in normal form. A vector-disturbance and the correction vector appear as terms in the right-hand side of the equation. For the realization of the method indicated it is necessary to know the maximum of the absolute value of the vector-disturbance and the phase vector of the process for a certain sequence of time instants. It is assumed that the right-hand side of the differential equation satisfies a Lipschitz condition in the phase coordinate.

1. The optimal course of a certain process is described by the vector differential equation

$$x^{\bullet} = f(x), \quad t \in [t_0, t'], \quad x(t_0) = x_0 \tag{1.1}$$

where vector x belongs to a finite-dimensional Euclidean space. It is assumed that the Lipschitz condition

$$|f(x') - f(x'')| \leqslant L\,|x' - x''|, \quad L \geqslant 0 \tag{1.2}$$

is satisfied in the domain being examined, owing to which the integral curve $x(t), t \in [t_0, t'], x(t_0) = x_0$ of Eq. (1.1) exists, is unique and is an absolutely continuous vector function (see [1]). In what follows the curve mentioned is called the unperturbed or optimal trajectory. Along the optimal trajectory the progress of the optimal process is impeded by a disturbance in the form of a measurable vector function $u^2(t), t \in [t_0, t']$ which appears in the right-hand side of Eq. (1.1) as a supplementary term. Besides what has been said about this disturbance, it is known only that it satisfies the constraint

$$|u^2(t)| \leqslant u_0^2, \quad t \in [t_0, t'] \tag{1.3}$$

To neutralize the effect of the disturbance indicated, we introduce a piecewise-constant vector-function $u^1(t), t \in [t_0, t']$ in the right-hand side of Eq. (1.1) as another term. As a result, from Eq. (1.1) we obtain the differential equation

$$y^{\bullet} = f(y) + u^1 + u^2, \quad t \in [t_0, t'], \quad y(t_0) = x_0 \tag{1.4}$$

whose integral curve $y(t), t \in [t_0, t'], y(t_0) = x_0$, also in an absolutely-continuous vector function, existing and unique on the interval being examined. In what follows this curve is called the perturbed trajectory.

Under the assumption that

$$f, \ L, \ x_0, \ t_0, \ t', \ u_0^2 \tag{1.5}$$

are known and, in addition, that the points $x(t_k)$ and $y(t_k)$ are known for a certain sequence $\{t_k\} \subset [t_0, t']$ of instants, we are required to find a method for constructing the function $u^1(t)$ which for any specified $\varepsilon > 0$ and any disturbance $u^2(t)$ (possessing the properties listed) ensures the satisfaction of the following inequality:

$$|x(t) - y(t)| \leqslant \varepsilon, \quad t \in [t_0, t'] \tag{1.6}$$

This method must include a rule for determining the sequence $\{t_k\}$ and must also take into account that under real conditions it takes some time to determine the position of points $x(t_k)$ and $y(t_k)$, and to calculate the vector function $u^1(t)$, $t \in [t_k, t_{k+1}]$

The solution of the problem of finding such a method, being a particular manifestation of the extrapolation method [2], is given below. The method proposed differs in particular from those in [3—5] (also in contrast to [4, 5], it is not necessary to know the probability characteristics of the disturbance to realize the method). We note that $\max_k(t_{k+1} - t_k) \nrightarrow 0$ as $\varepsilon \to 0$, therefore, when ε is fairly small, there is not enough time for actually carrying out the calculations.

2. The problem described admits of a more precise formulation in the form of the following antagonistic differential game of kind. This game is specified by the differential equation (1.4), where u^i $(i = 1, 2)$ is the i-th player's control satisfying all the requirement imposed, stated in Sect. 1; such a control is termed admissible. The first player's admissible strategies are piecewise-program strategies which associate with the quantities t_k, $x(t_k)$, $y(t_k)$ and (1.5) a number t_{k+1} and a vector function $u_k^1(t)$, $t \in [t_k, t_{k+1})$ being the restriction of a certain admissible control of the first player onto $[t_k, t_{k+1})$. The second player's admissible strategy can be any one satisfying the single requirement: the second player's control formed with its aid must be admissible (i.e., must satisfy the requirements mentioned in Sect. 1). In particular, we understand that there is discrimination against the first player in the game.

The i-th player's admissible strategy is denoted by v^i and the class of his admissible strategies, by V^i. The payoff in the game being analyzed is given in the following formula:

$$J(v^1, v^2) = \max_{t_0 \leqslant t \leqslant t'} |x(t) - y(t)| \tag{2.1}$$

where $x(t)$ is the unperturbed trajectory and $y(t)$ is the perturbed trajectory corresponding to the controls generated by strategies v^i.

Now the problem being solved in the present paper can be formulated as follows: describe constructively the strategy $v_0^1 \in V^1$, for which the following relation is satisfied:

$$J(v_0^1, v^2) \leqslant \varepsilon, \quad \forall\, v^2 \in v^2$$

From what has been said it is clear that in the game being analyzed the first player is the minimizing one and the second player the maximizing one (the latter player can be thought of as "Nature"). The sense of the game is for the first player to ensure a real course of the process (described by the perturbed trajectory) that does not differ, in the sense of criterion (2.1) by more than ε from the optimal course of the process (described by the unperturbed trajectory).

3. Assuming the satisfaction of the condition

$$u^1(t_k + \tau) = u_*^1 = \text{const}, \quad \tau \geqslant 0$$

by virtue of (1.2) and (1.3), from (1.4) we obtain

$$\frac{d}{d\tau}\rho(t_k, \tau) \leqslant \left|\frac{d}{d\tau} y(t_k + \tau)\right| \leqslant a_1 + L\rho(t_k, \tau)$$

$$\rho(t_k, \tau) = |y(t_k + \tau) - y(t_k)|, \quad \tau \geqslant 0$$

$$a_1 = |f(y(t_k)) + u_*^1| + u_0^2$$

for almost all τ. Hence it follows that the relation (see [6], p. 32):

$$\rho(t_k, \tau) \leqslant \rho_1(t_k, \tau) = a_1 L^{-1}(\exp(L\tau) - 1), \quad \tau \geqslant 0 \tag{3.1}$$

is satisfied for the solution $\rho_1(t_k, \tau)$ of the differential equation

$$\frac{d}{d\tau}\rho_1(t_k, \tau) - L\rho_1(t_k, \tau) - a_1 = 0, \quad \rho_1(t_k, 0) = 0, \quad \tau \geqslant 0$$

Integrating Eq. (1.4) along the trajectory $y(t)$, $t \in [t_k, t_k + \tau]$, $\tau > 0$, we obtain

$$y(t_k + \tau) - y(t_k) = \int_{t_k}^{t_k+\tau} f(y(t_k))\, d\theta + \tag{3.2}$$

$$\int_{t_k}^{t_k+\tau} [f(y(t_k + \theta)) - f(y(t_k))]\, d\theta + \int_{t_k}^{t_k+\tau} u_*^1\, d\theta + \int_{t_k}^{t_k+\tau} u^2(t_k + \theta)\, d\theta$$

By virtue of what has been said and of (3.1), the absolute value of the sum of the second and last terms in (3.2) does not exceed the quantity

$$R_1(t_k, \tau) = \int_0^\tau L\rho_1(t_k, \theta)\, d\theta + u_0^2\tau = a_1 L^{-1}(\exp(L\tau) - L\tau - 1) + u_0^2\tau \tag{3.3}$$

By $S(x, z)$ we denote a closed sphere in the Euclidean space being considered, with center at point x and radius z. From what we said above and from (3.2) and (3.3) follows the validity of

Lemma (on extrapolation). If $y(t_k)$ is a point on trajectory $y(t)$ of Eq. (1.4), corresponding to instant t_k, then for

$$u^1(t) = u_*^1, \quad t \in [t_k, t_k + \tau], \quad \tau > 0$$

and for any admissible control $u^2(t)$, $t \in [t_k, t_k + \tau]$ the point mentioned is transposed by the instant $t = t_k + \tau$ to the point $y(t_k + \tau)$ lying in a sphere of radius $R_1(t_k, \tau)$ of (3.3), with center at the point $y(t_k) + [f(y(t_k)) + u_*^1]\tau$, i.e.

$$y(t_k + \tau) \in S(y(t_k) + [f(y(t_k)) + u_*^1]\tau, \ R_1(t_k, \tau)) \tag{3.4}$$

Note. The radius of the sphere in (3.4) cannot be decreased in the general case. It is easy to establish this by considering an example with $L = 0$ and $u^2(t) = u_*^2 =$ const, where $|u_*^2| = u_0^2$. For this example the point $y(t_k + \tau)$ in (3.4) is located on the sphere's boundary, and, by appropriate choice of direction of vector u_*^2, at any point of the boundary. Consequently, in the general case the center of the minimum sphere containing all points of the form $y(t_k + \tau)$ is determined uniquely and coincides with the center of the sphere in (3.4).

4. Let us estimate from above the distance

$$r(t_k + \tau) = |x(t_k + \tau) - y(t_k + \tau)|, \quad \tau \geqslant 0$$

By applying the lemma on extrapolation in the particular case when $u^1(t) \equiv u^2(t) \equiv 0$ and, consequently, $y(t) \equiv x(t)$, we obtain the inclusion

$$x(t_k + \tau) \in S(x(t_k) + f(x(t_k))\tau, \ R(t_k, \tau)), \quad \tau \geqslant 0 \tag{4.1}$$
$$R(t_k, \tau) = |f(x(t_k))| L^{-1}(\exp(L\tau) - L\tau - 1)$$

From (3.4) and (4.1) follows

$$r(t_k+\tau) \leqslant r_1(t_k+\tau) \equiv |[y(t_k) - x(t_k)] + [f(y(t_k)) - f(x(t_k)) + u_*^1]\tau| + R_1(t_k, \tau) + R(t_k, \tau) \qquad (4.2)$$

It is easy to verify that $r_1(t_k+\tau)$ is a concave function of argument τ.

5. Let us generalize relations (3.4) and (4.2) to the case of control

$$u^1(t) = \begin{cases} 0, & t \in [t_k, t_k+\Delta\tau), \quad \Delta\tau > 0 \\ u_*^1 = \text{const}, & t \in [t_k+\Delta\tau, t_k+\Delta\tau+\tau], \quad \tau > 0 \end{cases} \qquad (5.1)$$

By virtue of the lemma on extrapolation we find (see (3.3))

$$y(t_k+\theta) \in S(y(t_k) + f(y(t_k))\theta, R'(t_k, \theta)) \qquad (5.2)$$
$$R'(t_k, \theta) = [|f(y(t_k))| + u_0^2] L^{-1}(\exp(L\theta) - L\theta - 1) + u_0^2\theta, \quad \theta \in [0, \Delta\tau]$$

By virtue of the same lemma we have

$$y(t_k+\Delta\tau+\varphi) \in S(y(t_k+\Delta\tau) + [f(y(t_k+\Delta\tau)) + u_*^1]\varphi, R_1(t_k+\Delta\tau, \varphi)), \quad \varphi \in [0, \tau] \qquad (5.3)$$

From (1.2) and (3.1) follows the inequality

$$|f(y(t_k+\Delta\tau)) - f(y(t_k))| \leqslant L\rho'(t_k, \Delta\tau) \qquad (5.4)$$
$$\rho'(t_k, \Delta\tau) = [|f(y(t_k))| + u_0^2] L^{-1}(\exp(L\Delta\tau) - 1)$$

A consequence of (5.2)—(5.4) is the inclusion

$$y(t_k+\Delta\tau+\varphi) \in S(y(t_k) + f(y(t_k))\Delta\tau + [f(y(t_k)) + u_*^1]\varphi, R_2(t_k, \Delta\tau, \varphi) + R'(t_k, \Delta\tau) + L\rho'(t_k, \Delta\tau)\varphi), \quad \varphi \in [0, \tau] \qquad (5.5)$$

where $R_2(t_k, \Delta\tau, \varphi)$ is obtained from $R_1(t_k+\Delta\tau, \varphi)$ of (3.3) by replacing in a_1 the quantity $|f(y(t_k+\Delta\tau)) + u_*^1|$ by the not lesser quantity (see (5.4))

$$|f(y(t_k)) + u_*^1| + L\rho'(t_k, \Delta\tau)$$

Keeping inclusions (4.1), (5.2) and (5.5) in mind, for the control (5.1) we obtain the following relations:

$$r(t_k+\theta) \leqslant r_1(t_k+\theta) \equiv |y(t_k) - x(t_k) + [f(y(t_k)) - f(x(t_k))]\theta| + R(t_k, \theta) + R'(t_k, \theta), \quad \theta \in [0, \Delta\tau] \qquad (5.6)$$

$$r(t_k+\Delta\tau+\varphi) \leqslant r_2(t_k+\Delta\tau+\varphi) \equiv |y(t_k) - x(t_k) + [f(y(t_k)) - f(x(t_k))](\Delta\tau+\varphi) + u_*^1\varphi| + R(t_k, \Delta\tau+\varphi) + R_2(t_k, \Delta\tau, \varphi) + R'(t_k, \Delta\tau) + L\rho'(t_k, \Delta\tau)\varphi, \quad \varphi \in [0, \tau] \qquad (5.7)$$

The concavity of function $r_2(t_k+\Delta\tau+\varphi)$ in the argument φ can be directly established as for the function $r_1(t_k+\theta)$; we can see that

$$r_1(t_k+\Delta\tau) = r_2(t_k+\Delta\tau)$$

6. The number F bounding the function $|f(y)|$ from above in the domain G_ε being examined

$$|f(y)| \leqslant F, \quad y \in G_\varepsilon$$

exists since by virtue of (1.2) the function $f(y)$ is continuous, while the set of points of G_ε, at a distance not exceeding ε from the unperturbed trajectory, is bounded. The number F can be computed in the following way. We specify some positive integer m and for the points

$$x_l = x(t_0 + l\Delta t), \quad \Delta t = (t' - t_0) / m, \quad l = 0, \ldots, m$$

of the unperturbed trajectory we construct the spherical neighborhoods $S(x_l, z_l)$, where

$$z_l = |f(x_l)| L^{-1} (\exp(L\Delta t) - 1)$$

Obviously, the neighborhoods constructed cover the whole unperturbed trajectory. Therefore, the maximum value of the function $|f(x)|$ on the unperturbed trajectory is no greater than the number $\max_l \{|f(x_l)| + Lz_l\}$; consequently, we can set

$$F = \max_{0 \leqslant l \leqslant m} \{|f(x_l)| + |f(x_l)| (\exp(L\Delta t) - 1)\} + \varepsilon L = \max_{0 \leqslant l \leqslant m} |f(x_l)| \exp(L\Delta t) + \varepsilon L \qquad (6.1)$$

We note, for example, that for the function $f(x) = Lx$ formula (6.1) yields in the limit as $m \to \infty$ the least upper bound of function $|f(y)|$ in domain G_ε as F. As regards the first term in the right-hand side of equality (6.1), it is evident that in the limit as $m \to \infty$ it yields, for any function $f(y)$ being examined, the least upper bound of $|f(x)|$ on the unperturbed trajectory.

7. Let numbers α and β satisfy the following conditions:

$$0 < \alpha < 1, \quad 0 < \beta < 1, \quad \alpha + 2\beta < 1 \qquad (7.1)$$

Lemma 7.1. If

$$r(t_k) \leqslant \varepsilon\alpha, \quad u^1(t) = 0, \quad t \geqslant t_k$$

(see Sect. 4), then

$$r(t_k + \tau) \leqslant \varepsilon(1 - \beta), \quad \tau \in [0, \tau_\alpha] \qquad (7.2)$$

$$\tau_\alpha = \varepsilon(1 - \alpha - \beta) / (L\varepsilon + u_0^2)$$

The lemma's validity follows from the fact that at almost every instant t the rate of displacement of point $y(t)$ relative to point $x(t)$ equals $[f(y(t)) - f(x(t)) + u^2(t)]$. Until the inequality $r(t) \leqslant \varepsilon$ is satisfied, the absolute value of the last expression, by virtue of (1.2) and (1.3), exceeds the denominator of the fraction in (7.2).

Lemma 7.2. If

$$r_1(t_k) \leqslant \varepsilon(1 - \beta), \quad u^1(t) = 0, \quad t \geqslant t_k \qquad (7.3)$$

then

$$r_1(t_k + \tau) \leqslant \varepsilon, \quad \tau \in [0, \tau_\beta]$$

$$\tau_\beta = \varepsilon\beta / [L\varepsilon + u_0^2 + {}^1/_2 (2F + u_0^2) LT \exp(LT)], \quad T = t' - t_0 \qquad (7.4)$$

As a matter of fact, from (5.6) and (7.3) we have

$$r_1(t_k + \tau) \leqslant \varepsilon(1 - \beta) + L\varepsilon\tau + R(t_k, \tau) + R'(t_k, \tau) \qquad (7.5)$$

Using Maclaurin's formula, we obtain

$$R(t_k, \tau) \leqslant {}^1/_2 |f(x(t_k))| L\tau^2 \exp(L\tau) \leqslant {}^1/_2 FLT \exp(LT)\tau \qquad (7.6)$$

$$R'(t_k, \tau) \leqslant {}^1/_2 [|f(x(t_k))| + u_0^2] L\tau^2 \exp(L\tau) + u_0^2 \tau \leqslant \qquad (7.7)$$

$$\tfrac{1}{2}(F + u_0^2) LT\exp(LT)\tau + u_0^2\tau, \quad \tau \in [t_k, t']$$

Hence it follows

$$R(t_k, \tau) + R'(t_k, \tau) \leqslant \{\tfrac{1}{2}(2F + u_0^2) LT\exp(LT) + u_0^2\}\tau \equiv W(\tau) \quad (7.8)$$

Having replaced in (7.5) the sum of the last two terms by the right-hand side of inequality (7.8), we equate the new right-hand side obtained for inequality (7.5) to the number ε. The root of the resulting equation relative to τ is the τ_β of (7.4). The conclusion in Lemma 7.2 follows from the method of finding the number τ_β The inequality

$$\tau_\alpha > \tau_\beta, \quad R(t_k, \tau_\beta) + R'(t_k, \tau_\beta) \leqslant W(\tau) < \varepsilon\beta \quad (7.9)$$

is a consequence of relations (7.1), (7.2), (7.4), (7.7) and (7.8) and of the method of finding the number τ_β.

8. Lemma 8.1. If

$$r(t_k) \in (\varepsilon\alpha, \varepsilon(1 - \beta)] \quad (8.1)$$

and $u_{1,k}^1$ and τ_k satisfy the system of relations

$$y(t_k) - x(t_k) + [f(y(t_k)) - f(x(t_k))](\tau_\beta + \tau_k) + u_{1,k}^1\tau_k = 0 \quad (8.2)$$
$$R(t_k, \tau_\beta + \tau_k) + R_2(t_k, \tau_\beta, \tau_k) + R'(t_k, \tau_\beta) + L\rho'(t_k, \tau_\beta)\tau_k = \varepsilon(1 - \beta)$$

then under the control

$$u_k^1(t) = \begin{cases} 0, & t \in [t_k, t_k + \tau_\beta) \\ u_{1,k}^1, & t \in [t_k + \tau_\beta, t_k + \tau_\beta + \tau_k] \end{cases} \quad (8.3)$$

the inequalities

$$r_1(t_k + \tau) \leqslant \varepsilon, \quad r(t_k + \tau) \leqslant \varepsilon, \quad \tau \in [0, \tau_\beta] \quad (8.4)$$

$$r(t_k + \tau_\beta + \tau_k) \leqslant \varepsilon(1 - \beta), \quad r(t_k + \tau_\beta + \tau) \leqslant \varepsilon, \ \tau \in [0, \tau_k] \quad (8.5)$$

are satisfied.

Proof. From (8.1) and (4.2) follows the inclusion

$$r_1(t_k) \in (\varepsilon\alpha, \varepsilon(1 - \beta)]$$

and inequality (8.4) follows from it from Lemma 7.2, and from (8.3) and (5.6). The inequality

$$r_2(t_k + \tau_\beta) \leqslant \varepsilon \quad (8.6)$$

is a consequence of (8.4), (5.6) and the equality $\Delta\tau = \tau_\beta$ in (5,1) and (8.3), while from conditions (8.2) and (5.7) ensues the equality

$$r_2(t_k + \tau_\beta + \tau_k) = \varepsilon(1 - \beta) \quad (8.7)$$

the consequence of which and of (5.7) is the first inequality in (8.5). Since the function $r_2(t_k + \tau_\beta + \tau)$, $0 \leqslant \tau \leqslant \tau_k$ is concave, from (8.6) and (8.7) follows the inequality

$$r_2(t_k + \tau_\beta + \tau) \leqslant \varepsilon, \quad \tau \in [0, \tau_k]$$

and from it, the second inequality in (8.5).

The existence of a solution of system (8.2) follows from a simple analysis of the system. We express $u_{1,k}^1$ from the first equation of the system and substitute it into the second equation of the system. The resultant left-hand side of the second equation we denote by $P(\tau_k)$; the equation itself takes the form

$$P(\tau_k) = \varepsilon(1 - \beta). \quad (8.8)$$

From (7.1) and (7.9) it follows that $P(0) < \varepsilon(1 - \beta)$. It is easy to note that $P(\tau)$

is a continuous function increasing unboundedly as $\tau \to \infty$. Therefore, the unique smallest root of Eq. (8.8) exists.

9. In final form the function $P(\tau)$ is expressed in terms of elementary functions, but this form is cumbersome; it is advisable to find a simpler equation whose solution would be analogous to the solution τ_k in the sense of Lemma 8.1. Using the formula (7.6) with subsequent transformations of type (7.7) we construct a linear increasing function $P_1(\tau)$ connected with $P(\tau)$ as follows:

$$P(\tau) \leqslant P_1(\tau), \quad \tau \geqslant 0; \quad P_1(0) < \varepsilon(1-\beta) \tag{9.1}$$

More precisely, we replace the terms of the left-hand side of Eq. (8.8) by the right-hand sides of the inequalities

$$R(t_k, \tau_\beta + \tau) \leqslant {}^1/_2\, FL\, T \exp(LT)(\tau_\beta + \tau)$$
$$R_2(t_k, \tau_\beta, \tau) \leqslant {}^1/_2\, \{F + [\varepsilon + L\varepsilon(\tau_\beta + T)]\}\, L\tau \exp(LT) + u_0{}^2\tau$$

Since in view of the first equation in (8.2)

$$\begin{aligned} &|u^1_{1,k}| \leqslant [\varepsilon + L\varepsilon(\tau_\beta + \tau_k)]\tau_k{}^{-1} \\ &R'(t_k, \tau_\beta) \leqslant {}^1/_2\,(F + u_0{}^2)\, L\tau_\beta{}^2 \exp(L\tau_\beta) + u_0{}^2\tau_\beta \\ &L\rho'(t_k, \tau_\beta)\,\tau \leqslant (F + u_0{}^2)\, L\tau_\beta \exp(L\tau_\beta)\,\tau \end{aligned} \tag{9.2}$$

We see that the function $P_1(\tau)$ constructed possesses all the required properties (the second inequality in (9.1) follows from (7.1) and (7.9)).

By τ^* we denote a root of the equation

$$P_1(\tau) = \varepsilon(1-\beta) \tag{9.3}$$

and by $u^1_{2,k}$ we denote the vector corresponding to it, obtained from the first equation in (8.2) solved relative to $u^1_{1,k}$ after substituting τ^* for τ_k. For what follows it is essential, as we see from the analysis carried out, that the number τ^* is unique and independent of k. The pair τ^* and $u^1_{2,k}$ is analogous to the pair τ_k and $u^1_{1,k}$ in the following sense: all assertions of Lemma 8.1 remain in force if in relations (8.2)—(8.5) we replace τ_k by τ^* and $u^1_{1,k}$ by $u^1_{2,k}$, while in the second equation of system (8.2), $P(\tau_k)$ by $P_1(\tau^*)$. As a matter of fact, relations (8.4) are preserved since neither the number τ_β nor the control $u^1(t)$ change in the interval $[t_k, t_k + \tau_\beta]$. In view of (5.7) and the fact that functions P and P_1 are connected by inequality (9.1), the function $r_2(t_k + \tau_\beta + \varphi)$ and the function $r_3(t_k + \tau_\beta + \varphi)$ obtained from r_2 by replacing the terms forming $P(\varphi)$, by the function $P_1(\varphi)$, are connected by the inequality

$$r_2(t_k + \tau_\beta + \varphi) \leqslant r_3(t_k + \tau_\beta + \varphi), \quad \varphi \geqslant 0$$

As is easy to note, the properties of function r_3 are similar to the properties of r_2, used to prove Lemma 8.1 (in particular, the equality $r_2(t_k + \tau_\beta) = r_3(t_k + \tau_\beta)$ is satisfied; therefore, the second part of the lemma — the relations (8.5) — goes over, when function r_2 is replaced by function r_3, into the relations resulting from (8.5) when τ_k is replaced by τ^*. We can note also that all assertions of Lemma 8.1 remain in force if τ^* is replaced by a number $\tau' \in (0, \tau^*)$; however, as we see from the estimate

$$|u^1_{2,k}| \leqslant [\varepsilon + L\varepsilon(\tau_\beta + \tau^*)] / \tau^* = \varepsilon / \tau^* + L\varepsilon\,(\tau_\beta / \tau^* + 1) \tag{9.4}$$

being a consequence of (9.2), the upper bound of the absolute value of the vector-correction increases in this case.

10. We can now describe the solution of the problem in the present paper. We find F (6.1); we specify numbers α and β satisfying conditions (7.1); we find the numbers τ_α (7.2) and τ_β (7.4); we compute the root τ^* of Eq. (9.3); we determine the strategy v_0^1 in the following way: at the instant t_k we set $u_k^1(t) = 0,\ t \in [t_k,\ t_k + \tau_\beta)$; we find $r(t_k)$; if $r(t_k) \leqslant \varepsilon\alpha$, we define $u_k^1(t) = 0,\quad t \in [t_k + \tau_\beta,\ t_{k+1}]$ and $t_{k+1} = t_k + \tau_\alpha$; if $r(t_k) \in (\varepsilon\alpha,\ \varepsilon(1-\beta)]$, then, having replaced τ_k in the first equality in (8.2) by τ^* and having replaced $u_{1,k}^1$ by $u_{2,k}^1$, we find $u_{2,k}^1$, after which we define $\bar{u}_k^1(t) = u_{2,k}^1,\ t \in [t_k + \tau_\beta,\ t_{k+1})$ and $t_{k+1} = t_k + \tau_\beta + \tau^*$. From everything we have said earlier it follows that the strategy constructed possesses all the properties required.

Notes. 1°. The computation of $r(t_k)$ and $u_{2,k}^1$ must be carried out on the time interval $[t_k, t_k + \tau_\beta]$.

2°. As τ^* we can take the number

$$\tau^\circ = \min\{\tau^*, \tau_\alpha - \tau_\beta\}$$

Then the sequence $\{t_k\}$, $t_k = t_0 + k(\tau_\beta + \tau^\circ)$ becomes known right away and, consequently, even before effecting the correction we can compute the vectors $x(t_k)$ and $f(x(t_k))$ (but, in this case, the right-hand side of inequality (9.4) can increase).

3°. If the constant L is not given, but the function $f(x)$ is sufficiently regular, we can estimate L by constructing the neighborhoods of points x_l considered in Sect. 6 and making use of the expansion of function $f(x)$ into Taylor series in a neighborhood of each point x_l.

On the basis of the method presented in this paper, a monitored calculation of a model example was carried out on an electronic computer. The block diagram of the calculation program was compiled using the description presented above.

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Translated by N. H. C.